James Ruse Agricultural High School

2022 **Year 12 TRIAL HSC EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total marks: 70

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–12)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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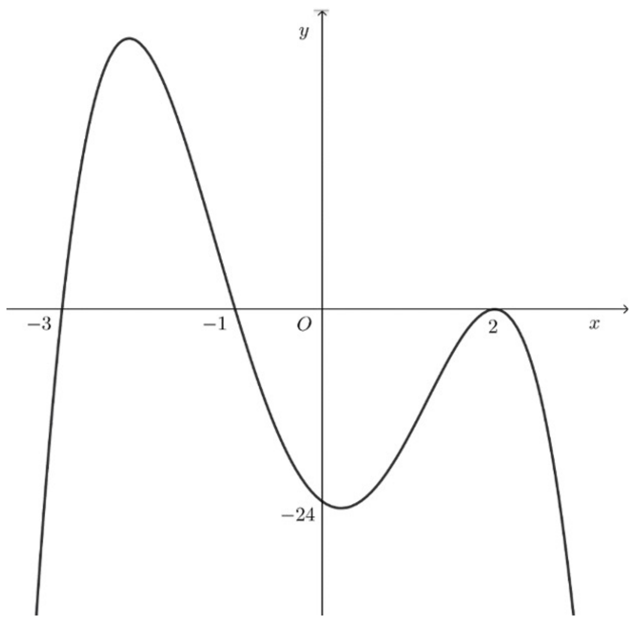
1 The diagram below shows the graph of $P(x) = a(x+b)(x+c)(x+d)^2$.



Which of the following is a possible set of values for a,b,c and d?

A. $-24, 3, 1, -2$

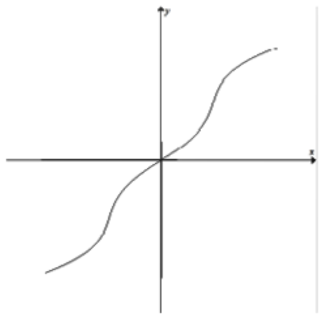
B. $-2, 3, 1, -2$

C. $-2, -1, -3, 2$

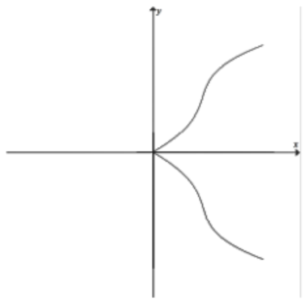
D. $-24, -1, -3, 2$

2 $f(x)$ is an odd function and $f(-1) < 0$. A possible graph showing $y = f(-|x|)$ is

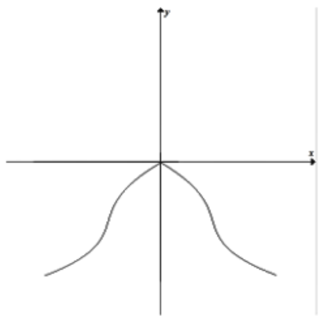
A.



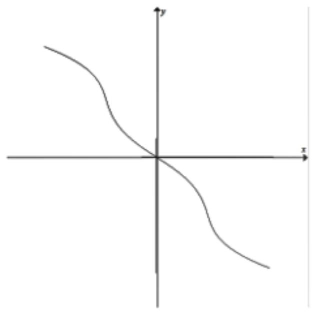
B.



C.



D.



3 Which is a correct anti-derivative for $\dfrac{6}{\sqrt{4-9x^2}}$?

A. $\sin^{-1}\left(\frac{3x}{2}\right)$

B. $2\sin^{-1}\left(\frac{3x}{2}\right)$

C. $3\sin^{-1}\left(\frac{3x}{2}\right)$

D. $6\sin^{-1}\left(\frac{3x}{2}\right)$

4 Consider two hot bodies, A and B, which have temperatures of 120°C and 80°C at $t = 0$ seconds. The temperature of the surrounding is 40°C. The ratio of the rate of cooling of body A to body B at $t = 0$ seconds will be:

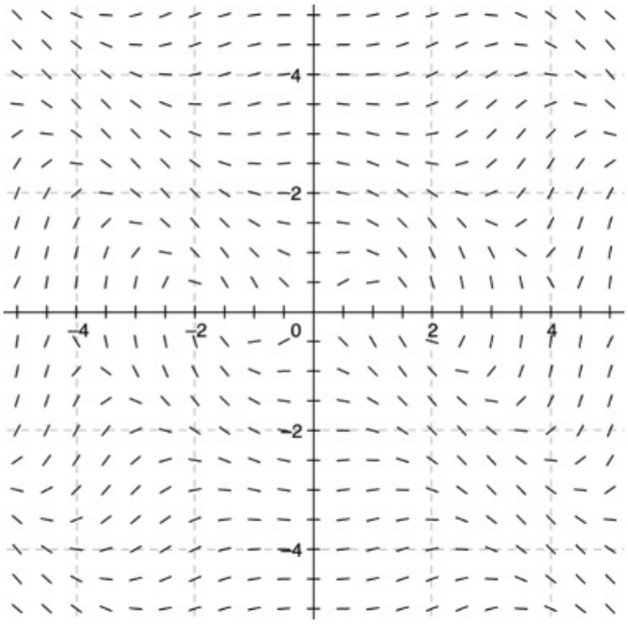
A. 1:2

B. 2:1

C. 3:2

D. 2:3

5 Which of the following differential equations yields the direction field below?



A. $\dfrac{dy}{dx} = \dfrac{x\cos(x+y)}{y}$

B. $\dfrac{dy}{dx} = \dfrac{y\cos(x+y)}{x}$

C. $\dfrac{dy}{dx} = \dfrac{x\sin(x+y)}{y}$

D. $\dfrac{dy}{dx} = \dfrac{y\sin(x+y)}{x}$

6 A particle has position vector given by $\underset{\sim}{r}(t) = \cos^{-1}(t)\,\underset{\sim}{i} + \sin^{-1}(t)\,\underset{\sim}{j}$. The distance travelled by the particle in the interval [-1,1] is closest to

A. 6

B. $\sqrt{3}\pi$

C. 5

D. $\sqrt{2}\pi$

7 Given $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$, which of the following is the simplified form of $\cos^{-1}(4x^3 - 3x)$?

A. $\pi - 3\sin^{-1}x$

B. $3\sin^{-1}x$

C. $3\cos^{-1}x$

D. $\pi - 3\cos^{-1}x$

8 How many integers do not satisfy the inequality $3|x-1| + x^2 - 7 > 0$?

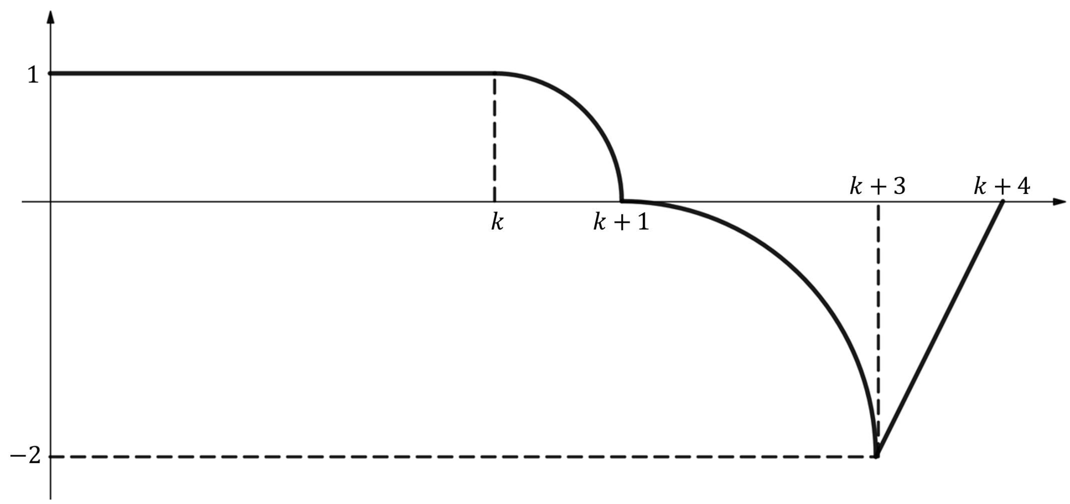
A. 4

B. 3

C. 2

D. 1

9 If $\int_0^{k+4} f(x)dx = 0$, what is the correct value of k?



A. $6 - \dfrac{\pi}{4}$

B. $5 - \dfrac{5\pi}{2}$

C. $6 - \dfrac{\pi}{2}$

D. $5 - \dfrac{5\pi}{4}$

10 The graph of $y = \dfrac{1}{a + bx + 4ax^2}$, where $a > 1$, has only one asymptote when

A. $-5 < b < 5$

B. $b \leq -4a$ or $b \geq 4a$

C. $-1 \leq b \leq 4$

D. $b < -4a$ or $b > 4a$

Section II

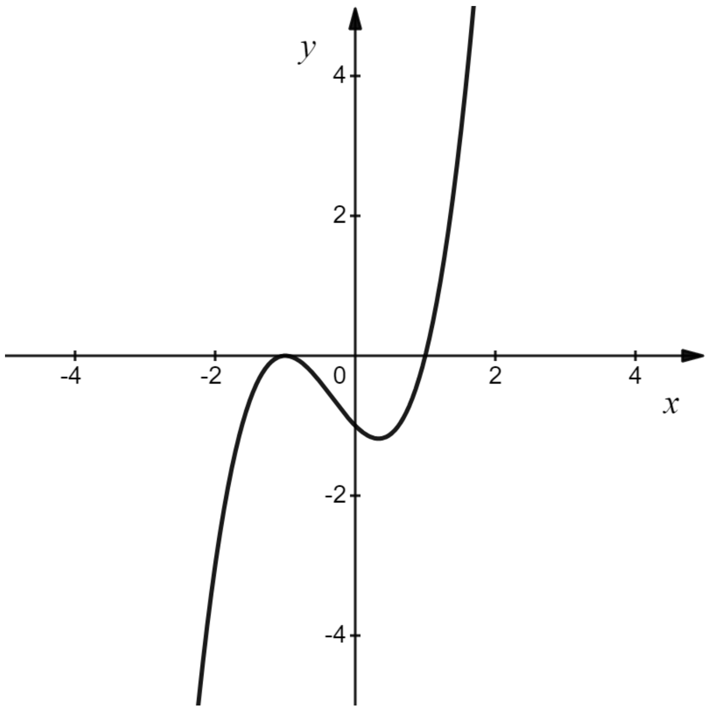
60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question on a new page. Extra writing pages are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new page

(a) Consider the graph of $f(x) = (x+1)^2(x-1)$ given below. **2**



Using the axes provided on the reverse of the Multiple Choice answer sheet, sketch the graph of $y^2 = f(x)$, showing all essential features.

(b) Use the substitution $u = e^{2x}$ to find **3**

$$\int \left(e^{e^{2x}}\right)^2 e^{2x}\,dx$$

Question 11 continues over the page

(c) Three forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, and $\overrightarrow{F_3}$ are represented by vectors $10\underset{\sim}{i}+20\underset{\sim}{j}$, $20\underset{\sim}{i}-30\underset{\sim}{j}$ and $-6\underset{\sim}{i}+20\underset{\sim}{j}$ Newtons respectively.

(i) Draw a sketch showing the relative magnitude and direction of the three forces. **2**

(ii) Determine the resultant force, expressed as a vector, and find its magnitude. **2**

(iii) An additional force $\overrightarrow{F_4} = k\underset{\sim}{j}$ is now applied so that the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, and $\overrightarrow{F_3}$ and $\overrightarrow{F_4}$ has magnitude 30N. What is the value of k? **2**

(d) You are asked to provide an 8-letter password which may only be comprised of the letters A, B and C.

(i) How many 8-letter passwords are there to choose from? **1**

(ii) How many of the 8-letter passwords contain only B's and C's? **1**

(iii) If you decide that your 8-letter password must use each of the letters A, B and C at least once, how many possible passwords are there to choose from? **2**

End of Question 11

Question 12 (15 marks) Start a new page

(a) The quartic polynomial $P(x) = x^4 + px^3 + qx^2 + px + 1$ has $x = \alpha$ as a double zero.

(i) Show that α^{-1} is also a zero of $P(x)$. **2**

(ii) Find the fourth root in terms of α. **1**

(iii) Hence show that $p^2 = 4(q - 2)$. **3**

(b) Suppose 3000 people are surveyed on political preference and asked the following question: **3**

"Would you vote for Party A in the next election, yes or no?"

If the actual probability of selecting someone in the affirmative is $p = 0.55$, find, using the normal distribution, the probability that the sample proportion $\hat{p}$ is within 2% of the actual value p.

You may use the z-score table provided.

(c)

(i) Show that the differential equation **2**

$$\frac{dy}{dx} = \frac{2xy}{x^2 - 1}$$

describes a family of parabolas with x-intercepts at $(1, 0)$ and $(-1, 0)$.

(ii) Now consider the differential equation **3**

$$\frac{dy}{dx} = \frac{1 - x^2}{2xy}$$

Find the equation of the curve which satisfies this differential equation and passes through the point $(1, 1)$. Express your answer as a function of x.

(iii) What can be said about the curve in part (ii) in relation to the family of curves in part (i)? **1**

End of Question 12

Question 13 (15 marks) Start a new page

(a) A student is given the chance to win 6^X dollars on two rolls of an unbiased, six-sided die, where X is the number of times a '3' is rolled.

(i) What is the most the student may win? **1**

(ii) How much is the student expected to win? Give your answer correct to two decimal places. **2**

(b) Solve for all $x \in \mathbb{R}$: $\sin x + \cos 2x - \sin 3x = 0$ **3**

(c) Consider the curve $y = 3\cos^{-1}\left(\frac{x}{2}\right)$.

(i) Sketch the graph of the curve $y = 3\cos^{-1}\left(\frac{x}{2}\right)$. **2**

(ii) The area bounded by the curve $y = 3\cos^{-1}\left(\frac{x}{2}\right)$, the y-axis and the lines $y = \frac{\pi}{2}$ and $y = \frac{5\pi}{2}$ is rotated around the y-axis to form an hourglass. Find the volume of the hourglass formed. **4**

(d) Prove by mathematical induction for all positive integers $n \geq 1$ that **3**

$$1^2 \times 2 + 2^2 \times 2^2 + 3^2 \times 2^3 + \ldots + n^2 \times 2^n = (n^2 - 2n + 3)2^{n+1} - 6$$

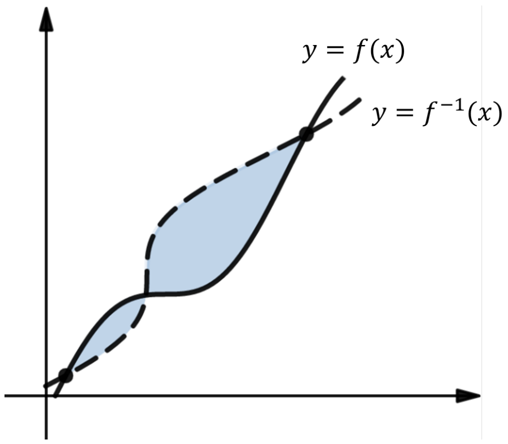
End of Question 13

Question 14 (15 marks) Start a new page

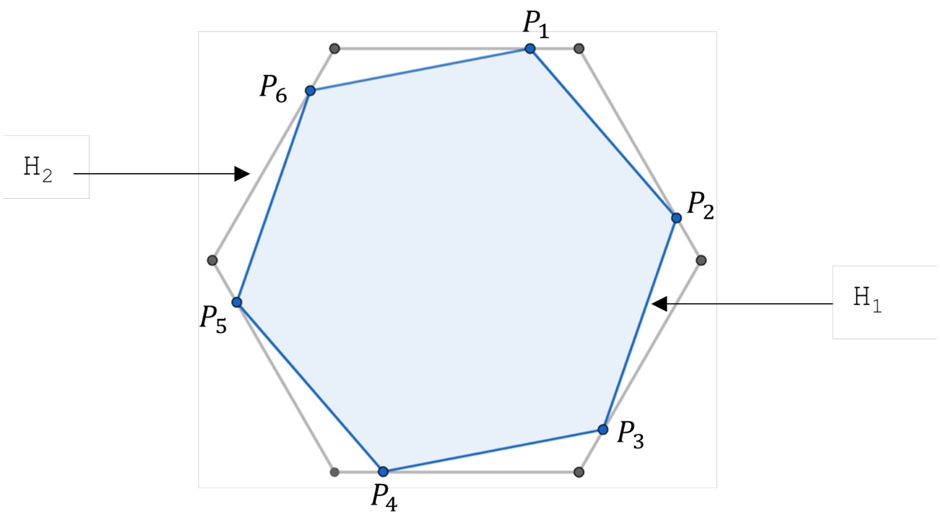
(a) Let $f(x) = \frac{1}{2}(2x - 1 + 2\sin x)$. The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are shown below.

(i) Find the points of intersection of the two curves. **1**

(ii) Find the shaded area enclosed by the two curves. **3**



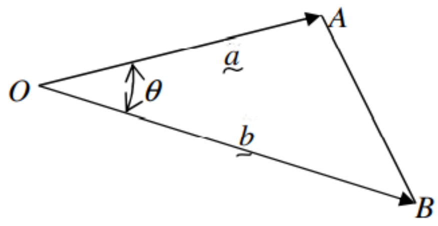
(b) Six points P_1, P_2, P_3, P_4, P_5 and P_6 form the vertices of a regular hexagon H_1. An animated program is rotating these six points at a constant speed of 3 cm/s around the perimeter of another regular hexagon, H_2, which has side length of 20 cm. **4**



Initially, the six vertices of H_1 lie on top of the six vertices of H_2. Determine the rate of change of the area of H_1 ten seconds after the animation begins.

Question 14 continues over the page

(c) Triangle OAB is defined by two non-parallel and non-zero vectors $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. The angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is θ. **3**



Express the area of triangle OAB in terms of dot products of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(d) Show that $\cos\left(\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right) + \cos^{-1} x\right) = 0$ for all real x, $-1 < x < 1$. **4**

You may assume that the range of $\dfrac{x}{\sqrt{1-x^2}}$ is all real numbers.

End of Question 14

End of Examination.

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MC.

1. B
2. C
3. B
4. B
5. A
6. D
7. C
8. A
9. D
10. C

Question 11

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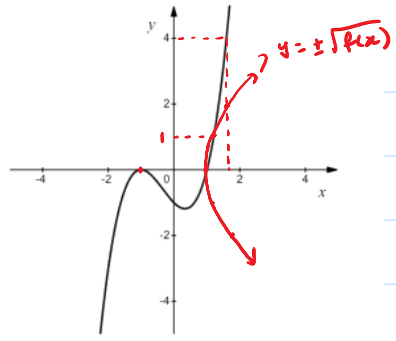
a)



1 mark for shape ($\sqrt{f(x)} > f(x)$ for $0 < y < 1$)

1 mark for accuracy (point at $x = -1$ AND either $(2, 3)$ on graph or correct point for when $y = 2$)

b) $u = e^{2x}$

$\frac{du}{dx} = 2e^{2x}$

$du = 2e^{2x}\,dx$

$\int \left(e^{e^{2x}}\right)^2 e^{2x}\,dx$

$= \frac{1}{2}\int \left(e^{e^{2x}}\right)^2 2e^{2x}\,dx$

$= \frac{1}{2}\int \left(e^{u}\right)^2 du$ ——— 1 mark for correct u substitution into integral

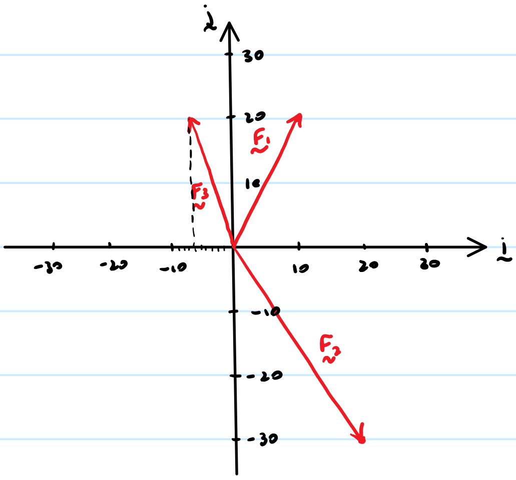
$= \frac{1}{2}\int e^{2u}\,du$

$= \frac{1}{2} \times \frac{1}{2} \times e^{2u}$ ——— 1 mark for correct antiderivative in terms of u

$= \frac{1}{4}e^{2u}$

$= \frac{1}{4}e^{2e^{2x}} + C$ ——— 1 mark for re-substituting back $u = e^{2x}$ into final answer

c)i)



1 mark for correct quadrants

1 mark for accuracy

Note: All vectors must be on the same diagram (ie one axis)

If drawn on separate diagrams 1 mark if all drawn on same scale.

c)ii) $\underset{\sim}{F}_{net} = 10\underset{\sim}{i} + 20\underset{\sim}{j} + 20\underset{\sim}{i} - 30\underset{\sim}{j} - 6\underset{\sim}{i} + 20\underset{\sim}{j}$

$= 24\underset{\sim}{i} + 10\underset{\sim}{j}$ (1) for net force

$|\underset{\sim}{F}_{net}| = \sqrt{24^2 + 10^2}$ (1) for magnitude

$= \sqrt{676}$

$= 26\ N$

iii) $24\underset{\sim}{i} + 10\underset{\sim}{j} + k\underset{\sim}{j}$

$= 24\underset{\sim}{i} + (k+10)\underset{\sim}{j}$

$\sqrt{24^2 + (k+10)^2} = 30$ (1) for correct setup

$24^2 + (k+10)^2 = 900$ (1) for correct values of k

$(k+10)^2 = 324$

$k + 10 = \pm 18$

$k = \pm 18 - 10$

$= -28, 8$

d)i) $3^8 = 6561$ (1) mark for correct answer

ii) $2^8 = 256$ (1) mark for correct answer

Also accepted interpretation of $2^8 - 2 = 254$ which contain only B's & C's but excludes all B's & all C's

iii) $3^8 - 3 \times 256 + 3 = 5796$

or

$3^8 - 3 \times 254 = 5796$

(2) marks for full correct answer

(1) for $3^8 - 3 \times 256 = 5793$

MATHEMATICS Extension 1 : Question 12...

Suggested Solutions	Marks Awarded	Marker's Comments
(i) $P(\alpha)=\alpha^4+p\alpha^3+q\alpha^2+p\alpha+1$ $=0$ $P(\frac{1}{\alpha})=\frac{1}{\alpha^4}+\frac{p}{\alpha^3}+\frac{q}{\alpha^2}+\frac{p}{\alpha}+1$ $=\frac{1+p\alpha+q\alpha^2+p\alpha^3+\alpha^4}{\alpha^4}$ $=0$ $\therefore P(\frac{1}{\alpha})$ is a root	1 for $P(\alpha)=0$ 1	• Some students went on incorrect root and made it hard for themselves • students lost $\frac{1}{\alpha^4}$ in their answer.
(ii) Product of roots $\alpha, \alpha, \frac{1}{\alpha}, \beta$ $\alpha^2\times\frac{1}{\alpha}\times\beta=1$ $\beta=\frac{1}{\alpha}$	1	• need to use the term without p or q • students using other terms struggled to find the root.
(iii) sum of the roots $\alpha+\alpha+\frac{1}{\alpha}+\frac{1}{\alpha}=-p$ $2(\alpha+\frac{1}{\alpha})=-p$	1	• some students failed to have negative sign
sum of the roots 2 at a time $\alpha^2+\alpha\times\frac{1}{\alpha}+\alpha\times\frac{1}{\alpha}+\alpha\times\frac{1}{\alpha}+\alpha\times\frac{1}{\alpha}+\frac{1}{\alpha^2}=q$ $q=\alpha^2+\frac{1}{\alpha^2}+4$	1	$\alpha^2+\frac{1}{\alpha^2}+2$ a common incorrect answer made it difficult to prove correctly

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks Awarded	Marker's Comments
$p^2 = 4\left(\alpha^2 + \frac{1}{\alpha^2} + 2\right)$ $= 4(q-2)$	1	expanding and equating coefficients could gain correct answer.
(b)		
$0.55 - 0.55 \times 0.02 \le \hat{p} \le 0.55 + 0.55 \times 0.02$ $0.539 \le \hat{p} \le 0.561$	1	• significant number of students just did $0.55 - 0.02$ and $0.55 + 0.02$ gave 0.9772 as answer
$6^2 = \frac{pq}{n} = \sqrt{0.0000825} = 0.0091$		
$z_1 = \frac{0.54 - 0.55}{0.0091} = -1.211$ $z_2 = \frac{0.56 - 0.55}{0.0091} = 1.211$	1	
using normal table $2P(0.8869 - 0.5)$ $= 0.7738$	1	

(3)

MATHEMATICS Extension 1 : Question 13		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
c) (i) $\frac{dy}{dx} = \frac{2xy}{x^2-1}$		well done
$\int \frac{dy}{y} = \int \frac{2x}{x^2-1} dx$		• substituting (1,0) to evaluate c ~~was~~ meant you were using what you were trying to prove
$\ln\|y\| = \ln\|x^2-1\| + c$	1	– students write
$\|y\| = e^{\ln\|x^2-1\|+c}$ $= e^c\|x^2-1\|$		$\|y\| = \|x^2-1\| + c$ which does give intercepts $(-1,0)$ and $(1,0)$
$y = A(x-1)(x+1)$ which is a family of parabolas with x-intercepts $(-1,0)$ and $(1,0)$	1	• students need to make sure they answer question asked.
(ii) $\frac{dy}{dx} = \frac{1-x^2}{2xy}$		
$\int 2y\,dy = \int \frac{1-x^2}{x} dx$		
$y^2 = \ln\|x\| - \frac{x^2}{2} + c$	1	
$(1,1)$ $1 = \ln 1 - \frac{1}{2} + c$ $c = \frac{3}{2}$	1	careless arithmetic errors.
$y^2 = \ln\|x\| - \frac{x^2}{2} + \frac{3}{2}$ through $x=1, y=1$ so $y = \sqrt{\ln\|x\| - \frac{x^2}{2} + \frac{3}{2}}$	1	needed to write as a function of x $y = \pm\sqrt{}$ is not a function

(4)

MATHEMATICS Extension 1 : Question 12		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(iii) $\frac{dy}{dx} \times \frac{dy}{dx} = -1$ the curve in part(ii) meets the family of parabolas in part (i) at right angles	1	Poorly done. • inverse functions was common incorrect answer. • Needed to mention right angle to gain mark.

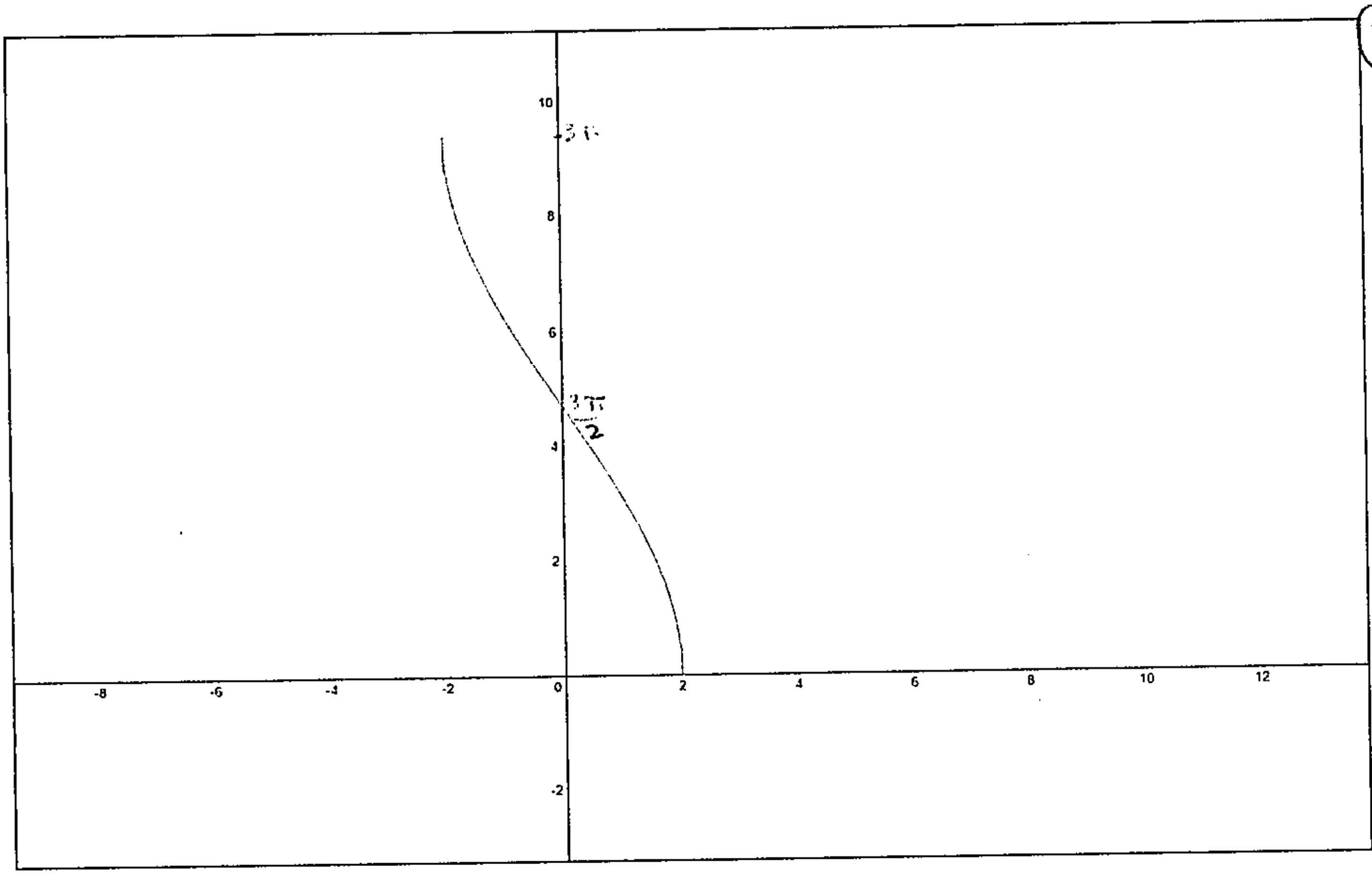
MATHEMATICS Extension 1 : Question 13

Suggested Solutions	Marks Awarded	Marker's Comments
a) i) $6^2 = 36$	1	well done
(ii) $p = \frac{1}{6}$ $E(6^x)$ $= 6^0\binom{2}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^2 + 6^1\binom{2}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^1 + 6^2\binom{2}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0$ $= \frac{1}{36}(25 + 60 + 36)$ $= \$3.36$	1 1	1 for binomial 2 for correct answer · poorly done most students used 0, 1, 2 not $6^0, 6^1, 6^2$ \$1.82 common incorrect answer. · students who showed binomial gained 1 mark
b) $\sin x + \cos 2x - \sin 3x = 0$ $\sin x - \sin 3x = 2\cos 2x \sin(-x)$ $\therefore \cos 2x - 2\cos 2x \sin x = 0$ $\cos 2x(1 - \sin x) = 0$ $\cos 2x = 0$ or $\sin x = \frac{1}{2}$	1 1	1 mark for correct manipulation of trig result 2 marks for gaining correct equation

MATHEMATICS Extension 2: Question 13

Suggested Solutions	Marks Awarded	Marker's Comments
b) $2x = (2k+1)\frac{\pi}{2}$ $\therefore$ $x = (2k+1)\frac{\pi}{4}$		needed to generate all solutions
$x = k\pi + (-1)^k \cdot \frac{\pi}{6} \quad k \in \mathbb{Z}$	1	needed $k \in \mathbb{Z}$. If write solutions needed negative as well.
c) see graph.		
(ii) $x = 2\cos\left(\frac{y}{3}\right)$		
$x^2 = 4\cos^2\frac{y}{3}$	1	
$V = \pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x^2\,dy + \pi \int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} x^2\,dy$		
$= 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 4\cos^2\frac{y}{3}\,dy$		
$= 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\left(1+\cos\frac{2y}{3}\right)dy$	1 for converting $\cos^2\frac{2y}{3}$	
$= 4\pi \left[y + \frac{3}{2}\sin\frac{2y}{3} \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$	1 for integration	solutions with incorrect limits needed to evaluate $\sin\frac{\pi}{3}$ to gain mark.
$= 4\pi \left(\frac{3\pi}{2} + \frac{3}{2}\sin\pi - \frac{\pi}{2} - \frac{3}{2}\sin\frac{\pi}{3} \right]$		
$= 4\pi \left[\pi - \frac{3\sqrt{3}}{4} \right]$	1	

③



1 mark for correct shape

- Many students made it look like a horizontal point of inflexion and did not gain mark

$y = 3\cos^{-1}\left(\frac{x}{2}\right)$

1 mark for correct position

- Students used poor scales which distorted graph because they were different on each axis.

④

MATHEMATICS Extension 1 : Question 13		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(d) $n=1$ LHS $= 1\times 2$ RHS $= (1-2+3)2-6$ $= 2$ $= 2$ $\therefore$ LHS = RHS so true for $n=1$	1	• must write LHS and RHS not as one long statement.
Prove true for $n=k$ $1^2\times 2 + 2^2\times 2^2 + \cdots + k^2\times 2^k = (k^2-2k+3)\times 2^{k+1}-6$ Assume true for $n=k+1$ $1^2\times 2 + \cdot\ k^2\times 2^k + (k+1)^2\times 2^{k+1}$ $= ((k+1)^2 - 2(k+1)+3)\times 2^{k+2} - 6$		
$\therefore 1^2\times 2 + \cdots k^2\times 2^k + (k+1)^2\, 2^{k+1}$ $= (k^2-2k+3)2^{k+1} - 6 + (k+1)^2\, 2^{k+1}$ (using assumption) $\doteqdot 2^{k+1}(k^2-2k+3+k^2+2k+1) - 6$ $= 2^{k+1}(2k^2+4) - 6$ $= 2^{k+1}(k^2+2) - 6$	1	well done. need to make it clear when you are using assumption

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MATHEMATICS X1 : Question……..		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
Now $RHS = (k^2+2k+1-2k-2+3)2^{k+2} - 6$ $= (k^2+2)^{2k+2} - 6$ $= LHS$ $\therefore$ true by process of mathematical induction for $n \geq 1$.		

Question 14

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a) i) Point of intersection of $f(x)$ & $f^{-1}(x)$ is the same as point of intersection of $f(x)$ & x

$$\frac{1}{2}(2x-1+2\sin x)=x$$

$$2x-1+2\sin x=2x$$

$$2\sin x=1$$

$$\sin x=\frac{1}{2}$$

$$x=\frac{\pi}{6}+2k\pi,\ \frac{5\pi}{6}+2k\pi \quad k\in\mathbb{Z}$$

or

$$x=\frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{13\pi}{6}$$

① mark for correct general solution or correct first 3 solutions

ii) $$A=2\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} f(x)-x\,dx+2\int_{\frac{5\pi}{6}}^{\frac{13\pi}{6}} x-f(x)\,dx$$

$$=2\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}(2x-1+2\sin x)-x\,dx+2\int_{\frac{5\pi}{6}}^{\frac{13\pi}{6}} x-\frac{1}{2}(2x-1+2\sin x)\,dx$$ ① for correct setup

$$=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2x-1+2\sin x-2x\,dx+\int_{\frac{5\pi}{6}}^{\frac{13\pi}{6}} 2x-(2x-1+2\sin x)\,dx$$

$$=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\sin x-1\,dx+\int_{\frac{5\pi}{6}}^{\frac{13\pi}{6}} 1-2\sin x\,dx$$

$$=\left[-2\cos x-x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}+\left[x+2\cos x\right]_{\frac{5\pi}{6}}^{\frac{13\pi}{6}}$$ ① for correct antiderivative

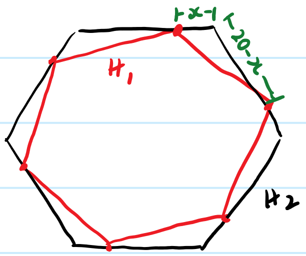
$$=\left(-2\cos\frac{5\pi}{6}-\frac{5\pi}{6}\right)-\left(-2\cos\frac{\pi}{6}-\frac{\pi}{6}\right)+\left(\frac{13\pi}{6}+2\cos\frac{13\pi}{6}\right)-\left(\frac{5\pi}{6}+2\cos\frac{5\pi}{6}\right)$$

$$=-2\times-\frac{\sqrt{3}}{2}-\frac{5\pi}{6}+2\times\frac{\sqrt{3}}{2}+\frac{\pi}{6}+\frac{13\pi}{6}+2\times\frac{\sqrt{3}}{2}-\frac{5\pi}{6}-2\times-\frac{\sqrt{3}}{2}$$

$$=\sqrt{3}-\frac{5\pi}{6}+\sqrt{3}+\frac{\pi}{6}+\frac{13\pi}{6}+\sqrt{3}-\frac{5\pi}{6}+\sqrt{3}$$

$$=4\sqrt{3}+\frac{2\pi}{3}$$ ① for evaluating correctly

b)



Let x be the distance each vertex on H_1 travels after t seconds

$\frac{dx}{dt} = 3$ cm/s

Let A = area of H_1

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

Let ℓ be the side length of H_1

$\ell^2 = x^2 + (20-x)^2 - 2 \times x(20-x)\cos 120°$

$= x^2 + 400 - 40x + x^2 - 2x(20-x)(-\frac{1}{2})$

$= x^2 + 400 - 40x + x^2 + 20x - x^2$

$= x^2 - 20x + 400$ ① for finding an expression for the side length of H_1

$A = 6 \times \frac{1}{2} \times \ell \times \ell \times \sin 60°$

$= \frac{3\sqrt{3}}{2}\ell^2$

$= \frac{3\sqrt{3}}{2}(x^2 - 20x + 400)$ ① for finding an expression for the area of H_1

$\frac{dA}{dx} = \frac{3\sqrt{3}}{2}(2x - 20)$

$= 3\sqrt{3}(x - 10)$

$\frac{dA}{dt} = 3\sqrt{3}(x-10) \times 3$

$= 9\sqrt{3}(x - 10)$

After 10s, the point has travelled $3 \times 10 = 30$cm, $30 - 20 = 10$ so it is 10cm from the vertex

ie need to sub in $x = 10$ ① for finding correct value of x to sub in

$\frac{dA}{dt} = 9\sqrt{3}(10 - 10)$

$= 0$ cm²/s ① for finding the correct expression for $\frac{dA}{dt}$ and evaluating correctly when $x = 10$.

c) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$ — ①

$A = \frac{1}{2}|\underset{\sim}{a}||\underset{\sim}{b}|\sin\theta$

$2A = |\underset{\sim}{a}||\underset{\sim}{b}|\sin\theta$ — ②

$①^2 + ②^2 \quad (\underset{\sim}{a}\cdot\underset{\sim}{b})^2 + (2A)^2 = |\underset{\sim}{a}|^2|\underset{\sim}{b}|^2\cos^2\theta + |\underset{\sim}{a}|^2|\underset{\sim}{b}|^2\sin^2\theta$

$= |\underset{\sim}{a}|^2|\underset{\sim}{b}|^2(\cos^2\theta + \sin^2\theta)$ ① mark for using Pythagorean identity

$= |\underset{\sim}{a}|^2|\underset{\sim}{b}|^2$

$4A^2 = |\underset{\sim}{a}|^2|\underset{\sim}{b}|^2 - (\underset{\sim}{a}\cdot\underset{\sim}{b})^2$

$2A = \sqrt{|\underset{\sim}{a}|^2|\underset{\sim}{b}|^2 - (\underset{\sim}{a}\cdot\underset{\sim}{b})^2} \quad (A>0)$

$A = \frac{1}{2}\sqrt{|\underset{\sim}{a}|^2|\underset{\sim}{b}|^2 - (\underset{\sim}{a}\cdot\underset{\sim}{b})^2}$ ① mark for simplifying to this expression

$= \frac{1}{2}\sqrt{(\underset{\sim}{a}\cdot\underset{\sim}{a})(\underset{\sim}{b}\cdot\underset{\sim}{b}) - (\underset{\sim}{a}\cdot\underset{\sim}{b})^2}$ ① mark for final expression only in dot products

$A = \frac{1}{2}\underset{\sim}{a}\cdot\underset{\sim}{b}\tan\theta$

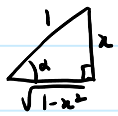
$A = \frac{1}{2}|\underset{\sim}{a}||\underset{\sim}{b}|\sin\left(\cos^{-1}\left(\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}\right)\right)$

} both got ① mark

d) Let $\alpha = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$, $\quad \beta = \cos^{-1}x$

$\tan\alpha = \frac{x}{\sqrt{1-x^2}} \qquad x = \cos\beta$

When $0<x<1$ and $0<\alpha<\frac{\pi}{2}$



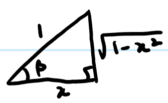
$\cos\alpha = \sqrt{1-x^2}$

$\sin\alpha = x$

① mark for drawing triangles correctly & fully labeled.

① mark for considering all cases, not just the acute cases

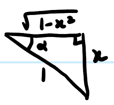
when $0<x<1$ and $0<\beta<\frac{\pi}{2}$



$\cos\beta = x$

$\sin\beta = \sqrt{1-x^2}$

when $-1<x<0$ and $-\frac{\pi}{2}<\alpha<0$



$\cos\alpha = \sqrt{1-x^2}$

$\sin\alpha = x$

when $-1<x<0$ and $\frac{\pi}{2}<\beta<\pi$

$\cos\beta = x, \sin\beta = \sqrt{1-x^2}$

$\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \cos^{-1}x\right) = 0 \quad$ for $\quad x\in(-1,1)$

$\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)\cos(\cos^{-1}x) - \sin\left(\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)\sin(\cos^{-1}x)$ ① mark for expanding using compound angles

$(\sqrt{1-x^2})x - x\sqrt{1-x^2} = 0$ for all cases ① mark for showing that the expression $=0$